Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

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MONETARY ECONOMICS: MACRO ASPECTS
SOLUTIONS TO JUNE 12 EXAM, 2015

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In a flex-price world with cash-in-advance constraints on investment and consumption, a positive inflation rate is optimal as it encourages capital accumulation.

A **False**. A cash-in-advance constraint invokes a marginal cost on the spending in question, which increases with the nominal interest rate, and thus the inflation rate by the Fisher relationship. The marginal cost only vanishes with a zero nominal interest rate, which will be the optimal monetary policy in this case as it will leave capital accumulation undistorted. This policy implementing the Friedman rule will imply a rate of deflation equal to the real interest rate.

(ii) A central bank operating under "strict inflation targeting" should never respond to the output gap.

A **False**. There can be cases where the output gap provides relevant information about the current state of the economy, and thus future inflation in economies where changes in demand and the output gap affect inflation with some lag. Even if the central bank does not care at all about the output gap *per se*, an observed increase in the output gap should therefore be met with an increase in the nominal interest rate as this will dampen the future effects on inflation. The output gap thus has the function of an "intermediate target" for the central bank.

(iii) In the simple New Keynesian model with goods-price rigidities, the optimal rate of inflation is zero because this maintains households' purchasing power.

A **False**. Positive or negative growth in goods prices would be matched by an equivalent growth rate in wages, leaving the real wage unchanged on average. A positive or negative inflation, on the other hand, have welfare costs in the New Keynesian model, as it will distort relative demand among the consumption goods in the economy. The distortion arises as there will always be firms that do not adjust their prices. The optimal allocation of goods in the model is one where there is equal consumption of all goods; in effect requiring that relative prices are the same among any pair of goods. This is only feasible under sticky prices if inflation is zero.

QUESTION 2:

Money-in-the-utility-function and money demand

Consider an infinite-horizon economy in discrete time, where utility of the representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t), \qquad 0 < \beta < 1, \tag{1}$$

with

$$u(c_t, m_t) \equiv b \ln c_t + (1-b)\left(m^F \ln m_t - m_t\right), \qquad 0 < b < 1, \quad m^F > 0.$$

Agents maximize utility subject to the budget constraint

$$\begin{aligned} c_t + k_t + m_t &= f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{1}{1+\pi_t} m_{t-1}, \qquad 0 < \delta < 1 \\ &\equiv \omega_t, \end{aligned} \tag{2}$$

where c_t is consumption, m_t is real money balances at the end of period t, k_{t-1} is physical capital, τ_t are monetary transfers from the government, and π_t is the inflation rate. Function f satisfies $f' > 0$, $f'' < 0$.

(i) Derive the relevant first-order conditions for optimal behavior [Hint: Set up the value function $V(\omega_t) = \max_{c_t, m_t} \{u(c_t, m_t) + \beta V(\omega_{t+1})\}$ and substitute out ω_{t+1} by (2) and k_t by $k_t = \omega_t - c_t - m_t$.] Interpret the first-order conditions intuitively.

A Using the hint, one recovers the following first-order conditions:

$$\begin{aligned} u_c(c_t, m_t) &= \beta V'(\omega_{t+1})\left[f_k(k_t) + 1 - \delta\right], \\ u_m(c_t, m_t) + \beta V'(\omega_{t+1})\frac{1}{1+\pi_{t+1}} &= \beta V'(\omega_{t+1})\left[f_k(k_t) + 1 - \delta\right]. \end{aligned}$$

The first condition states consumption is chosen such that its marginal benefit (in terms of its marginal utility) equals its marginal cost (in terms of the discounted marginal value cost of lower next-period wealth times the real interest rate).

The second condition states real money balances are chosen such that their marginal benefits (in terms of marginal utility and discounted marginal value gain of higher next-period wealth corrected by inflation) equal their marginal cost (in terms of the discounted marginal value cost of lower next-period wealth times the real interest rate).

(ii) Show that the first-order conditions can be combined into

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t}{1+i_t}, \tag{3}$$

where $f_k(k_t) + 1 - \delta = (1+i_t)/(1+\pi_{t+1})$ defines i_t as the nominal interest rate. Discuss (3) and explain whether steady-state superneutrality holds in the model.

A Combining the two conditions gives

$$\begin{aligned} \frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} &= \frac{\beta V'(\omega_{t+1})\left[f_k(k_t) + 1 - \delta - \frac{1}{1+\pi_{t+1}}\right]}{\beta V'(\omega_{t+1})\left[f_k(k_t) + 1 - \delta\right]} \\ &= 1 - \frac{1}{\left[f_k(k_t) + 1 - \delta\right](1+\pi_{t+1})} \\ &= 1 - \frac{1}{\left[(1+i_t)/(1+\pi_{t+1})\right](1+\pi_{t+1})} \\ &= \frac{i_t}{1+i_t}, \end{aligned}$$

where the next-to-last line uses the definition of the nominal interest rate.

In this model, there will be steady-state superneutrality, as any change in the nominal interest rate only affects money holdings for given consumption. Real money holdings do not affect the marginal utility of consumption (or the marginal product of capital), so the model's steady state for capital is given by $1/\beta = f_k(k^{ss}) + 1 - \delta$ which is independent of monetary factors (the determination of the steady state capital stock can be shown formally by using the result from the Envelope Theorem, $V'(\omega_t) = \beta V'(\omega_{t+1})[f_k(k_t) + 1 - \delta]$, but a verbal explanation suffices).

(iii) Apply the particular functional form of u and characterize the monetary policy that maximizes the utility of the representative agent and find the corresponding optimal steady-state real balances. Explain the results intuitively.

A With the particular utility function, equation (3) gives the following steady-state relationship:

$$\frac{(1-b)\left(m^F/m^{ss} - 1\right)}{b/c^{ss}} = \frac{i^{ss}}{1+i^{ss}}.$$

As monetary policy "only" affects m^{ss}, the welfare-maximizing policy is one that induces $u_m(c_t, m_t) = 0$, which is equivalent to $i^{ss} = 0$. I.e., the Friedman rule where the opportunity cost of money balances is zero. In this case, it implies $m^{ss} = m^F$, where m^F is the optimum quantity of money in the model.

QUESTION 3:

Consider the following "New-Keynesian" log-linear model of a closed economy:

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(\widehat{i}_t - \mathrm{E}_t \pi_{t+1}\right) + u_t, \qquad \sigma > 0, \tag{1}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \qquad 0 < \beta < 1, \quad \kappa > 0, \tag{2}$$

$$\widehat{i}_t = \phi \pi_t + \varepsilon_t, \qquad \phi > 1, \tag{3}$$

where x_t is the output gap, $\widehat{i}_t$ is the nominal interest rate's deviation from steady state, and π_t is goods-price inflation, u_t is a mean-zero i.i.d. shock and ε_t is a mean-zero, i.i.d. "policy shock". E_t is the rational-expectations operator conditional upon all information up to and including period t.

(i) Explain in words how (1) and (2) can be derived from a micro-founded model, and explain the monetary transmission mechanism.

A Equation (1) is the "dynamic IS curve", which as a foundation uses a log-linearization of consumers' consumption-Euler equations: A lower real interest rate, $\widehat{i}_t -$ $\mathrm{E}_t\{\pi_{t+1}\}$, make consumers increase current consumption relative to future consumption. With consumption, c_t, being equal to output, y_t, application of the definition of the output gap as the difference between output at

flex-price output, leads to (1). The shock u_t will represent expected changes in the flex-price output.
Equation (2), the "New-Keynesian Phillips Curve", is derived from the optimal price-setting decisions of monopolistically competitive firms that operate under price stickiness of the Calvo form. Prices are set as a mark up over marginal costs, and as the output gap is proportional to marginal costs, it enters in (2) positively. Expected future prices are central for price determination, as firms are forward looking, since they acknowledge that the price set today may be in effect for some periods.
Monetary policy is transmitted onto the economy in the following way: A reduction in the nominal interest rate reduces the real interest rate (as prices are sticky), which then reduces consumption for given expected future consumption, and thus output and the output gap. This transmits into lower marginal costs, and thus lower prices set by firms (who can reset prices); as a result inflation goes down.

(ii) Derive the solutions for x_t and π_t. [Hint: Conjecture that the solutions are linear functions of ε_t and u_t, and use the method of undetermined coefficients.] Comment on the role of the policy parameter ϕ in terms of the output gap's and inflation's dependence on ε_t and u_t.

A Follow the hint, and conjecture the following solutions for the output gap and inflation:

$$\begin{aligned} x_t &= -A_\varepsilon \varepsilon_t + A_u u_t, \\ \pi_t &= -B_\varepsilon \varepsilon_t + B_u u_t. \end{aligned}$$

Forwarding these conjectures one period, take period-t expectations, give

$$\begin{aligned} \mathrm{E}_t x_{t+1} &= -A_\varepsilon \mathrm{E}_t \varepsilon_{t+1} + A_u \mathrm{E}_t u_{t+1} = 0, \\ \mathrm{E}_t \pi_{t+1} &= -B_\varepsilon \mathrm{E}_t \varepsilon_{t+1} + B_u \mathrm{E}_t u_{t+1} = 0, \end{aligned}$$

where the last equalities in both lines follow from the assumptions about the shocks; $\mathrm{E}_t \varepsilon_{t+1} = \mathrm{E}_t u_{t+1} = 0$. We then insert the conjectures and these expectations into the model (1)–(3), where (3) has been substituted into (1)

$$\begin{aligned} -A_\varepsilon \varepsilon_t + A_u u_t &= -\sigma^{-1} \left[\phi \left(-B_\varepsilon \varepsilon_t + B_u u_t \right) + \varepsilon_t \right] + u_t, \\ -B_\varepsilon \varepsilon_t + B_u u_t &= \kappa \left(-A_\varepsilon \varepsilon_t + A_u u_t \right). \end{aligned}$$

As these equations must hold for any ε_t, u_t, we differentiate w.r.t. these shocks on the left- and right-hand sides to obtain

$$\begin{aligned} -A_\varepsilon &= \sigma^{-1}\phi B_\varepsilon - \sigma^{-1}, \\ A_u &= -\sigma^{-1}\phi B_u + 1, \\ B_\varepsilon &= \kappa A_\varepsilon \\ B_u &= \kappa A_u \end{aligned}$$

We can then determine the coefficients as

$$\begin{aligned} -A_\varepsilon &= \sigma^{-1}\phi\kappa A_\varepsilon - \sigma^{-1} \\ A_\varepsilon &= \frac{\sigma^{-1}}{1+\sigma^{-1}\phi\kappa}, \\ B_\varepsilon &= \frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\phi\kappa} \end{aligned}$$

and

$$\begin{aligned} A_u &= -\sigma^{-1}\phi\kappa A_u + 1 \\ A_u &= \frac{1}{1+\sigma^{-1}\phi\kappa}, \\ B_u &= \frac{\kappa}{1+\sigma^{-1}\phi\kappa}. \end{aligned}$$

The solutions for the output gap and inflation are therefore

$$\begin{aligned} x_t &= -\frac{\sigma^{-1}}{1+\sigma^{-1}\phi\kappa}\varepsilon_t + \frac{1}{1+\sigma^{-1}\phi\kappa}u_t, \\ \pi_t &= -\frac{\sigma^{-1}\kappa}{1+\sigma^{-1}\phi\kappa}\varepsilon_t + \frac{\kappa}{1+\sigma^{-1}\phi\kappa}u_t. \end{aligned}$$

One can see that a higher inflation response, ϕ, in the interest-rate rule leads to a milder absolute effect of either shock on the output gap and inflation.
Consider the case of $\varepsilon_t > 0$. This is a case where the interest rate is hit by a positive shock. All things equal, this will increase the real interest rate and put downward pressure on the output gap and inflation. When the central bank responds to the drop in inflation by lowering the interest rate more than one-for-one, as $\phi > 1$, it will dampen the increase in the real interest rate and thus the effects of the shock on output gap and inflation. The stronger the response, the milder is the effect on the real interest rate, and thus the milder are the effects on the economy.
Consider the case of $u_t > 0$. This is a case of a direct shock to the output gap, which rises and implies an increase in inflation. As for the effects of the

central bank's response, the story is the same as for the interest-rate shocks, but with the signs reversed: When the central bank responds to the increase in inflation by raising the nominal interest rate more than one-for-one ($\phi > 1$), it will increase the real interest rate and thus dampen the expansionary effects of the shock on output gap and inflation. The stronger the response, the stronger is the effect on the real interest rate, and thus the milder are the effects of the shock on the economy.

(iii) Examine whether the parameter ϕ can be chosen such that the output gap and inflation are stabilized completely. Discuss whether such a situation is desirable, and whether its potential attainment is realistic.

A In the case where the central bank reacts extremely aggressive towards inflation changes, $\phi \to \infty$, the output gap and inflation will be completely insulated from the shocks. In the context of the underlying model this is desirable, as the central bank then achieves an elimination of the distortions associated with nominal rigidities. Fluctuations in the output gap are associated with inefficient fluctuations in firms' desired mark up, and fluctuations in inflation are synonymous with inefficient allocations across consumption goods.
In this case, the economy is characterized by the "divine coincidence" where shocks do not pose a trade off for monetary policy. This is a potential special case, which may not be realistic. If the economy is hit by other shocks that pose a trade off, a policy of exclusively aiming for inflation stability, which $\phi \to \infty$ can be interpreted as reflecting, will not be desirable. It will cause all impacts of the shocks to be on the output gap. (Also, a policy rule with such extreme responses could create volatility if inflation is not measured accurately.)